Numer. Math. Theor. Meth. Appl. Vol. **18**, No. 2, pp. 521-543
doi: 10.4208/nmtma.OA-2024-0123 May 2025

Noise Robust Physics-Informed Generative Adversarial Networks for Solving Stochastic Differential Equations

Lin Wang[1], Min Yang[1,*], Ruisong Gao[2] and Chuanjun Chen[1]

[1] *School of Mathematics and Information Sciences, Yantai University, Yantai 264005, China*
[2] *College of Science, China University of Petroleum East China, Qingdao 266580, China*

Received 24 October 2024; Accepted (in revised version) 22 January 2025

Abstract. This paper proposes a class of physics-informed neural networks called noise robust physics-informed generative adversarial networks (NR-PIGANs) to solve stochastic differential equations in the presence of noisy measurements. In these scenarios, while the governing equations are known, only a limited number of sensor measurements of the system parameters are available, and some may contain significant measurement errors. To address this, NR-PIGAN incorporates an additional noise generator with specific distribution constraints into a physics-informed generative adversarial network framework. The noise generator is trained alongside the clean data generators in an end-to-end manner, enabling the model to effectively capture both clean and noisy data distributions under the given physical constraints. Numerical experiments demonstrate that NR-PIGAN excels in handling forward and inverse problems under diverse noise perturbations, and its advantage becomes more pronounced as the noise level increases.

AMS subject classifications: 35Q68, 68T07, 68W25

Key words: Stochastic differential equation, inverse problem, noisy measurement, physics-informed, generative adversarial network.

1. Introduction

Stochastic differential equations (SDEs) [10] have emerged as pivotal mathematical tools for simulating the evolution of stochastic processes and have found increasingly widespread applications across diverse disciplines in recent years. By integrating differential equation theory with stochastic process theory, these equations effectively

*Corresponding author. *Email addresses:* wanglin8120@163.com (L. Wang), yang@ytu.edu.cn (M. Yang), grs1108@163.com (R. Gao), cjchen@ytu.edu.cn (C. Chen)

capture random variations in complex systems. As such, they are well-suited for simulating real-world processes such as financial market fluctuations [26], particle dispersion [33], and neural behavior [35].

In recent years, the rapid development of deep learning technology has made it an effective method for solving stochastic equations [14,23,27,29,30,34,39]. Particularly noteworthy is the emergence of the physics-informed neural network (PINN) paradigm [6,9,15,16,18,28,36,38], which seamlessly integrating physical insights as adaptable constraints within the loss function. This innovative approach utilizes machine learning techniques such as automatic differentiation [3] and gradient descent [2] for efficient training. Li *et al.* [21] introduced a physics-informed Karhunen-Loève method combined with neural networks for parameter estimation in SDEs under sparse measurements. Ma *et al.* [25] proposed a PINN method for solving time-dependent stochastic fractional PDEs. Teng *et al.* [31] developed a deep learning-based numerical algorithm to solve forward-backward doubly SDEs, even in high-dimensional cases. Additionally, Guo *et al.* [13] proposed a data-driven approach for SDEs using a normalized field flow-based method. In situations where exact analytical parameter representations are not available and data from sparse sensors is limited, Zhong *et al.* [40] utilized variational autoencoders to address forward, inverse, and mixed problems. Further, a closely related work to our research is the introduction of physics-informed generative adversarial networks (PI-WGAN) by Yang *et al.* [37]. This approach effectively combines generative adversarial networks (GANs) [11] with physical insights to tackle challenges in solving SDE problems through adversarial training for both the generator and discriminator. Recently, Gao *et al.* [7] introduced physics-informed variational embedding generative adversarial networks (PI-VEGAN) to enhance training stability by integrating an encoder that better captures the real data distribution. Furthermore, Gao *et al.* [8] also proposed physics-informed generator-encoder adversarial networks (PI-GEA) with latent space matching to enhances accuracy and stability without increasing computational costs. The above studies primarily consider ideal situations where sensor measurements are clean and accurate. However, in real-world scenarios, sensor measurements are often subject to noise, which can degrade model performance and therefore requires special handling.

There is a lot of research on the application of neural networks to solve SDEs under noisy conditions. For instance, Wang and Yao [32] introduced the vNPs-SDE model, which effectively addresses noisy, irregularly sampled data while providing robust uncertainty estimates. This model employs variants of neural processes to manage noisy in-distribution data, enabling more effective processing of the completed in-distribution data with SDE-Net [20]. Bonneville and Earls [4] proposed using Bayesian neural networks to recover system states and parameters from noisy measurement data related to underlying equations. They utilized Hamiltonian Monte Carlo to sample the posterior distribution of a deep Bayesian neural network and generated derivative datasets as surrogates for the system response. Their approach applies sequential threshold Bayesian linear regression on these derivatives to recover the original equation parameters. Liu *et al.* [24] developed a Bayesian physics-informed extreme learning machine