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Sparse Wavelet Element Method for Piezoelectric Equations in an Unbounded Domain

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Abstract. This paper is concerned with devising an efficient numerical method for the piezoelectric equations in an unbounded domain, which plays a fundamental role in design and analysis of microacoustic devices with piezoelectric substrate. We make use of the perfectly matched layer method to transform the underlying problem as a surrogate in a bounded domain, which is further solved by a sparse wavelet element method. The latter method can be viewed as a combination of a wavelet element method and a sparse grid method. The numerical results are performed to show the proposed method is very efficient and outperforms the usual finite element method. It can be naturally extended to two/three dimensional problems in an unbounded domain whose boundary consists of line segments or rectangles parallel to coordinate lines or planes.

AMS subject classifications: 65D40, 65T60, 65N22

Key words: Piezoelectric equation, sparse grid method, sparse tensor product method, wavelet element method.

1. Introduction

Microacoustic devices with piezoelectric substrate have been widely used in the field of modern communication and network of physical devices (IoT) [25, 42]. The work principle of all these microacoustic devices is governed by the so-called positive and reverse piezoelectric effect [11, 12, 29], that means, when a piezoelectric material is mechanically deformed, a voltage is generated inside the material, and conversely electric field can deform the piezoelectric material. According to this effect, electrical signals can propagate along the piezoelectric substrate in the interconversion of acoustic waves and electric fields. In many cases, the size of the piezoelectric substrate is much larger than the wavelength of the acoustic wave. Therefore, the mathematical

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model of the microacoustic devices always rely on the piezoelectric equations defined in an unbounded domain. In this paper, we are intended to design an efficient numerical method for such a problem.

For this purpose, the first issue to be settled is that the underlying problem is defined in an unbounded domain. As far as we know, there are several ways to address this difficulty [43]. The first class of methods are the so-called boundary integral equation (BIE) methods, which transform a partial differential equation in an unbounded domain as a boundary integral equation on the boundary in terms of Green's functions, and further develop numerical methods [13, 17, 20, 34]. The spectral methods are also frequently used to solve problems in unbounded domains because some of their basis functions, such as Hermite basis functions and Laguerre basis functions, are directly defined in unbounded domains [35–37, 39]. The other class of methods first approximate the problem in an unbounded domain by its bounded analogue through the absorbing boundary conditions (ABC) or the perfectly matched layer (PML) methods. After truncation, one can numerically solve the resulting bounded problems using the fundamental numerical methods, such as the finite element methods (FEMs), the wavelet element methods (WEMs) and the finite difference methods (FDMs). We refer the reader to [1, 15, 16, 31] for the ABC methods and [3, 26, 27, 33] for the PML methods. This paper will use the PML method in [22] to truncate the unbounded domain to transform the original problem into a problem in a bounded domain. It is worth noting that the PML methods have been widely borrowed to solve many kinds of problems involving microacoustic devices with piezoelectric substrate, such as the surface acoustic wave (SAW) and the bulk acoustic wave (BAW) resonators [22, 23, 32].

Next, we should focus on developing fast solvers for the underlying problem in a bounded domain. As is well known, the FEMs are a class of typical methods for numerically solving the piezoelectric equations in a bounded domain [2, 6, 22–24, 32]. However, there are two issues to be overcome for the methods. On the one hand, since the piezoelectric equations require to solve both the displacement and the electric potential, the storage overhead is very large for the underlying large scale linear system. On the other hand, because of the use of the PML approach, one needs to solve a large scale complex linear system, which is more difficult to deal with than a real linear system.

In regular domains, the sparse grid methods, also known as the sparse tensor product method, are well accepted approaches to devising efficient algorithms, which can get over the above two difficulties. The concept of these methods was first used by Smoljak to construct multivariate quadrature formulas [38]. Its main ideas include multiscale orthogonal (or biorthogonal) decomposition of nested vector spaces to create wavelet spaces, and then selecting a small number of wavelet bases from the full tensor product space to form a sparse tensor product space by some rules. Up to now, the sparse grid method has been applied to resolve several mathematical physics problems. For example, it was combined with the discontinuous Galerkin method to solve higher dimensional elliptic equations [41] as well as radiative transfer equations [19]. Some other applications can be found in [21, 44]. These works fully demonstrate the